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М. Н. Кирсанов, д-р физ.-мат. наук, **Сунь Цзясюань,** студент
Национальный исследовательский университет "МЭИ"
Тел.+7(495)362-73-14; E-mail: c216@ya.ru

НИЖНЯЯ ГРАНИЦА ЧАСТОТЫ СОБСТВЕННЫХ КОЛЕБАНИЙ КОНСОЛИ ФЕРМЫ МАНИПУЛЯТОРА

Ферма манипулятора, рассчитанного на динамические нагрузки, представляет собой плоскую статически определимую конструкцию с массами, распределенными по узлам консоли. Предлагается алгоритм вывода зависимости первой частоты колебаний конструкции от числа панелей в аналитической форме. Используются операторы системы символьной математики Maple и метод индукции по двум параметрами фермы: числу панелей в консоли и числу панелей в вертикальной опорной стойке - ферме. Коэффициенты искомой формулы находятся как общие члены последовательностей коэффициентов, полученных в результате решения серии задач с увеличивающимся числом панелей в ферме. Используется формула Донкерлея. Решение имеет высокую точность. Для сравнения использовался традиционный численный метод решения задачи.

Ключевые слова: *консоль, ферма, метод Донкерлея, колебания, первая частота*

M. N. Kirsanov, Sun Jiaxuan

LOWER BOUND OF THE NATURAL OSCILLATION FREQUENCY OF THE MANIPULATOR TRUSS CONSOLE

The truss of a manipulator designed for dynamic loads is a flat, statically determinate structure with masses distributed over the console nodes. An algorithm is proposed for deriving the dependence of the first oscillation frequency of the structure on the number of panels in an analytical form. The operators of the Maple symbolic mathematics system and the induction method are used for two parameters of the truss: the number of panels in the console and the number of panels in the vertical support rack-truss. The coefficients of the desired formula are found as common terms of the sequences of coefficients obtained as a result of solving a series of problems with an increasing number of panels in the truss. The formula of Dunkerley is used. The solution has high accuracy. For comparison, the traditional numerical method of solving the problem is used.

Keywords: *console, truss, Dunkerley method, oscillations, first frequency*

1. Introduction

The calculation of the natural oscillations frequencies for manipulators designed for high-speed work with high overloads and stresses of individual parts is of great importance. If the design of the manipulator truss contains many elements, then the calculation of the entire spectrum of oscillations becomes quite difficult. Numerical methods in such cases tend to accumulate rounding errors, which sometimes leads to unpredictable results. In fact, the entire spectrum for practice does not always need to be calculated. The main value here is the first, lowest frequency. To calculate the lower bound of this quantity, the Dunkerley method is known, which actually replaces the complex problem of the eigenvalues of a matrix (sometimes of very large dimension) with a simple calculation of its trace. This problem can also be solved analytically. In this paper, this method is used to derive a formula for the frequency of the plane model of the manipulator.

The solution is based on the method previously used in determining analytical expressions for the deflection of trusses [1]. The problems of determining the oscillation frequencies of beam trusses were solved in the Maple system in [2–5].

2. Dunkerley's approximation

The truss consists of a vertical (support) part with *m* panels in height and a console with masses fixed in the nodes (Fig. 1). The forces in the truss rods required to calculate the rigidity of the structure using the Mohr's integral are calculated in analytical form according to the program [1], written in the language of symbolic mathematics Maple.

To do this, the coordinates of the nodes and the grid diagram are entered into the program. The scheme is encoded by special lists N[i] containing the numbers of the ends of the rods. The rods and nodes are numbered (Fig. 2).

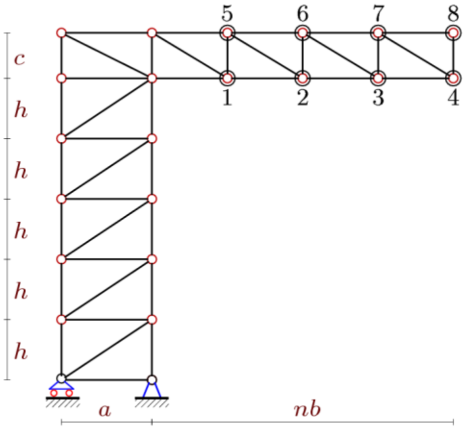

Figure 1. Truss, *m*=5, *n*=4

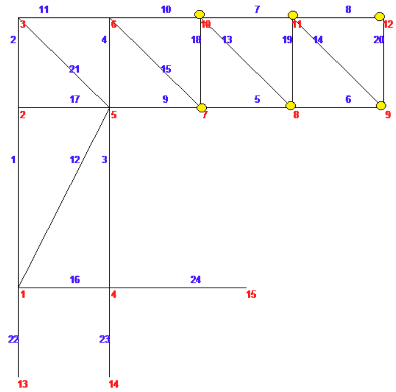
Figure 2. Numbering of nodes and rods, *m*=2, *n*=3.

Here is a fragment of the data entry program. The origin is placed in the leftmovable support:

```
H:=m*h-h+c:
for i to m do  x[i]:=0:x[i+m+1]:=a:
                y[i]:=h*i-h;y[i+m+1]:=h*i-h; end:
x[m+1]:=0:  y[m+1]:=H: x[2*m+2]:=a: y[2*m+2]:=H:
for i to n do  x[i+2*m+2]:=i*b+a;   y[i+2*m+2]:=H-c;
               x[i+2*m+n+2]:=i*b+a; y[i+2*m+n+2]:=H;  end:
```

Next, we consider the case $a=b$. The rods of the support posts are entered in two lists:

```
for i to m do
 N[i]:=[i,i+1]; N[i+m]:=[i+m+1,i+m+2];
end:
```

The total number of rods in the truss is $\eta = 4(m+n+1)$ rods, including three rods that model supports. The number of degrees of freedom of the system under the assumption that the masses move only vertically is $K=2n$. The solution according to the Dunkerley method [7] for the first oscillation frequency is expressed in terms of the oscillation frequencies of individual loads:

$$\omega_D = \sqrt{1/\sum_{p=1}^{K} 1/\omega_p^2}\,, \tag{1}$$

where ω_p is the oscillation frequency of the mass μ located in the console node. The equation of the motion of a single mass has the form:

$$\mu \ddot{y}_p + d_p y_p = 0,\ p=1,\ \dots,\ K,$$

where is d_p the stiffness coefficient y_p – is the mass displacement, and $\ddot{y}_p$ is the acceleration. Hence, the oscillation frequency of a single load (partial frequency) has the form: $\omega_p = \sqrt{d_p / \mu}$. The coefficient of rigidity is calculated using the Mohr integral:

$$\delta_p = 1/d_p = \sum_{j=1}^{\eta-3} \left(\tilde{S}_j^{(p)}\right)^2 l_j / (EF).$$

Here it is indicated: $\tilde{S}_j^{(p)}$ — the forces in the rod with the number *j* from the action of a single vertical force applied to the node where the mass with the number *p* is located, *E* – the elastic modulus of the rod material, *F* — the cross-sectional area of the rods. The cross sections and the material of the rods (except for the three supporting ones) are assumed to be the same for the entire truss. From (1) follows:

$$\omega_D^{-2} = \mu \sum_{p=1}^{K} \delta_p = \mu \Delta_n. \quad (2)$$

General view of the solution for the coefficient Δ_n at $m = 3$:

$$\Delta_n = \frac{C_1 a^3 + C_2 c^3 + C_3 d^3 + C_4 c^2 h}{c^2 EF}. \quad (3)$$

Solving the problem sequentially for $n = 1, 2, 3,...$, we get:

$$\Delta_1 = (4a^3 + 5c^3 + 20c^2 h + 4d^3)/(c^2 EF),$$
$$\Delta_2 = 2(12a^3 + 9c^3 + 36c^2 h + 8d^3)/(h^2 EF),$$
$$\Delta_3 = \left(80a^3 + 43c^3 + 172c^2 h + 40d^3\right)/\left(h^2 EF\right),$$
$$\Delta_4 = 4\left(50a^3 + 21c^3 + 84c^2 h + 20d^3\right)/\left(h^2 EF\right),...$$

To generalize these expressions to n, we use the **rgf_findrecur** operator from the special package **genfunc** of the Maple system. Thus, we obtain the recurrent equations for the elements of the sequences. For the coefficient C_1, for example, we have a linear homogeneous equation of the fifth order: $C_{1,n} = 5C_{1,n-1} - 10C_{1,n-2} + 10C_{1,n-3} - 5C_{1,n-4} + C_{1,n-5}$. The **rsolve** operator gives a solution to this equation:

$$C_1 = (n+1)^2 (n+2)/3.$$

Other coefficients are found in the same way:

$$C_2 = n(7n^2 + 6n + 7)/3,$$
$$C_3 = 2n(n+1)(n+2)/3,$$
$$C_4 = 4n(2n^2 + 6n + 7)/3.$$

The task contains two independent integer-valued parameter. To generalize the solution to an arbitrary number of panels in height *m*, you need to repeat the entire solution sequentially for different *m* =1, 2, … Calculations show that only the coefficient C_4 depends on the number of panels m . By induction, using the **rgf_findrecur** and **rsolve** operators, we obtain in the general case:

$$C_{4,n} = 2(m-1)n(2n^2+6n+7)/3.$$

Hence, taking into account (2) and (3), we obtain the final formula for the lower bound of the first natural frequency of oscillations of the truss:

$$\omega_D = c\sqrt{\frac{EF}{C_1a^3+C_2c^3+C_3d^3+C_4c^2h}}\ . \qquad (4)$$

3. Numerical verification

The accuracy of the obtained formula can be estimated by comparing it with a full-scale numerical calculation of all frequencies of the structure. The forces calculation can be performed in the same program that was used to calculate the forces in the analytical form.

The equations of mass μ motion have the form:

$$\mu\mathbf{I}_K\ddot{\mathbf{Y}}+\mathbf{D}_K\mathbf{Y}=0\,,$$

Here the following symbols are introduced: $\mathbf{D}_K$ – stiffness matrix, $\mathbf{Y}=[y_1,y_2,...,y_K]^T$ – vector of vertical displacements of loads, $\mathbf{I}_K$ – unit matrix, $\ddot{\mathbf{Y}}$ – vector of accelerations of nodes with masses μ. The stiffness matrix $\mathbf{D}_K$ is calculated as the inverse of the flexibility matrix $\mathbf{B}_K$, whose elements are determined using the Mohr's integral:

$$b_{i,j} = \sum_{p=1}^{\eta-3} S_p^{(i)}S_p^{(j)}l_p/(EF). \qquad (5)$$

Here $S_p^{(i)}$ — the force in the rod with the number p from the action of the unit vertical force at the node i, l_p — the length of the rod. The three rods in the supports are not deformed. The forces in these rods are not included in the sum (5).

The eigenvalues of the matrix $\mathbf{B}_K$ are found using the **Eigenvalues** operator from the specialized linear algebra package **LinearAlgebra** of the Maple system. The graph (Fig. 3) shows the curves of the dependence of the first frequency ω_{num}, obtained numerically, and ω_D according to the formula (4). The curve of the analytical solution according to Dunkerley is located slightly below the numerical solution, the curves are almost identical. Accepted: m = 3, $EF = 1000H$, μ = 100kg, a = 3m, h = 5 m. To clarify the nature of the relative error, you can trace the dependence of the value $\varepsilon = (\omega_{num}-\omega_D)/\omega_{num}$ from the number of panels.

The error of the solution varies depending on the number of panels from 0.92% for n = 2 to 0.68% for a large number of panels (Fig. 4).

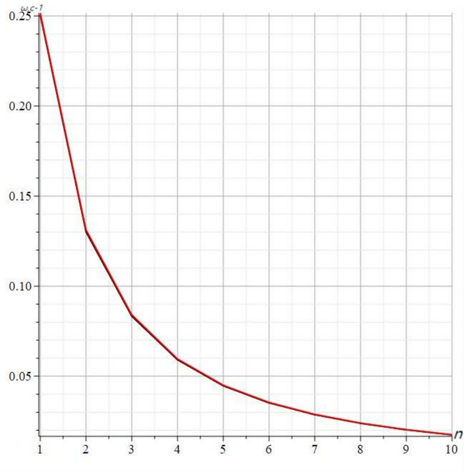

Figure 3. Dependence of the first frequency on the number of panels.

The resulting formula can be used to estimate the frequency of oscillations of the truss with a very large number of rods. As is known, the accuracy of the numerical calculation decreases with an increase in the number of structural elements, while the analytical solution at $n > 10$ has an almost constant and high accuracy.

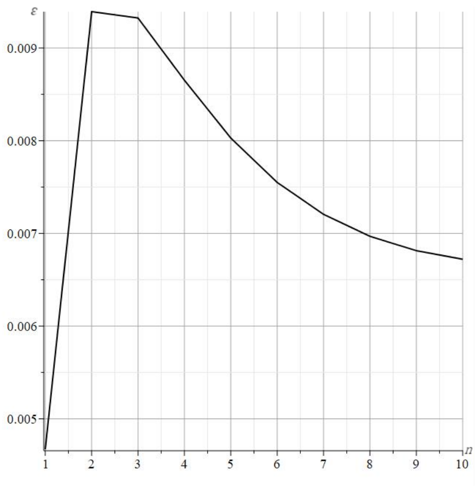

Figure 4. The error estimation of Dunkerley depending on the number of panels.

Numerical calculations and calculations using the formula (4) show that the oscillation frequency depends non-linearly on the height of the panel h (Fig. 5). The graphs are constructed according to the analytical solution (4) for $m = 8$ and the same values of the masses and stiffness of the rods as the previous graphs. As the number of n panels in the console increases, the extremum on the chart decreases and shifts to the right on the chart.

4. Conclusions

The method of estimating the first frequency by Dunkerley in the problems of analyzing the vibration frequencies of structures is rarely used in practice. This is due to the fact that for systems with a small number of degrees of freedom, its accuracy is low (29% according to [6]), and in the case of systems with a large number of degrees of freedom, numerical counting is necessary for its use, and the meaning of using the Dunkerley method is lost. Numerically, it is not difficult to calculate with sufficient accuracy all the frequencies of natural oscillations.

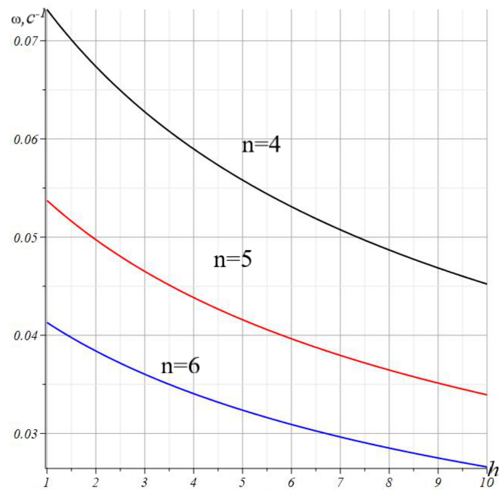

Figure 5. Dependence of the oscillation frequency on the height of the truss.

In this paper, the method of induction by two independent parameters for the derivation of analytical dependencies and the use of symbolic mathematics operators allowed us to obtain an analytical solution. As it turned out, the frequency dependence on the number of panels has a fairly compact shape and high accuracy. The formula is convenient for use in practical calculations, and the applied algorithm can be used in solving other similar problems for regular systems.

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